SIF8015 Logic

Exercise 1
Informal Statement Calculus

Task 1

Translating natural language sentences into logical expressions can be very tricky, since natural language sentences often are ambiguous, depend on the context they are in, and require some external knowledge to be interpreted. It is therefore not uncommon for several alternative twists to be equally acceptable (even though they might not be logically equivalent), depending on what you think is actually meant and on the "world" you intend to describe. Furthermore, several different choices of how to decompose a sentence into statement variables might be equally acceptable, depending on the purpose of the logical formalization of the sentence. In this exercise there will typically be 3-5 primitive statements in each compound statement.

Translate into symbols the following compound statements:

1. If n is an even integer and m is an odd integer, then n^2m^2 will be positive or zero.
2. Your sorting algorithm will run in $O(n \log n)$ time on an average problem of size n if you have implemented insertsort with a binary search or quicksort.
3. The ratio between a rational number x and an integer y is either a rational number or undefined.
4. Your new disk drive will never rotate with the speed of light, unless you are Superman and there is no green cryptonite in the immediate vicinity, in which case it will when you spin it around by hand.
5. "This statement is false" is either a true statement or a false statement, or it is not a statement at all.
6. Not only will your computer run a billion times faster than before and never crash, but when you install Microsoft's new operating system, you will also get a free lunch or hell will freeze over.

Task 2

A Norwegian children's song contains the statements "min hatt den har tre kanter" and "har den ei tre kanter, så er den ei min hatt". Translate into symbols these two compound statements. Are the statements equivalent? Show why or why not.

Task 3

Do exercises 3c, 3g, 3d, 5b, 5d, 6c, 6d and 7 on page 10 in the textbook by Hamilton.

Task 4

Do exercise 11a on page 15 in the textbook by Hamilton.

Truth-preserving transformations should be written like this:
$\mathcal{A}_1 \equiv$
$\mathcal{A}_2 \equiv$
$\mathcal{A}_3$

Task 5

Do exercise 13d on page 19 in the textbook by Hamilton.

Hint: Note that the negation of a statement in disjunctive normal form is a statement in conjunctive normal form.
And vice versa: The negation of a statement in conjunctive normal form is a statement in disjunctive normal form.

Task 6

Do exercise 18 on page 22 in the textbook by Hamilton.

Task 7

Do exercise 21 on page 26 in the textbook by Hamilton.

Hint: You can use propositon 1.32 in Hamilton or proof by "reductio ad absurdum".